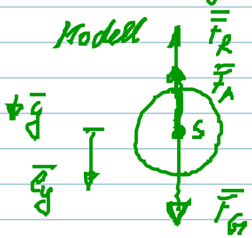
Lösung Kugelfallviskosimeter

Modell



IS: $m\ddot{\bar r} = \sum{}' \bar F^{(a)} + \sum{}' \bar Z$

$\bar Z = \bar 0$

$\bar F^{(a)}$: $\bar F_G = mg \cdot \bar e_y = V_K \rho_K g \cdot \bar e_y$

$\bar F_A = -V_K \rho_{Fl} \cdot g \cdot \bar e_y$

$\bar F_R = -k_d \cdot v \cdot \bar e_y$

$\bar e_x$: $0 = 0$

$\bar e_y$: $m\ddot y = V_K \cdot \rho_K g - V_K \rho_{Fl} \cdot g - k_d v$ $\quad v = \dot y$

Dgl.: $m\ddot y + k_d \dot y = g V_K (\rho_K - \rho_{Fl}) = F_0$

NR: k_d

$\bar F_R = -p A_K c_w \frac{\bar v}{|\bar v|}$

Vor.: Kugelströmung $1 < Re < 1000$ $\quad c_w = \frac{24}{Re}$

$Re = \frac{\rho_{Fl}}{\eta} \cdot d \cdot v$

$F_R = \underbrace{\frac{\rho_{Fl}}{2} v^2}_{p} \cdot \frac{\pi}{4} d^2 \cdot \frac{24\eta}{\rho_{Fl} \cdot d \cdot v} = \underbrace{3\pi\eta d \cdot v}_{\text{Gesetz von Stokes}} = k_d \cdot v$

$k_d = 3\pi\eta d$

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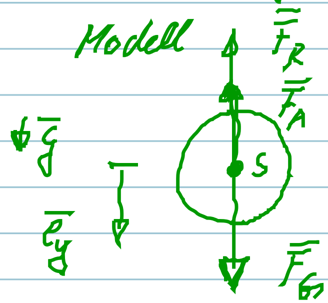

IS: $m\ddot{\vec r} = \sum \vec F^{(a)} + \sum \vec Z$

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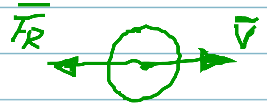
$\vec e_x$: $0 = 0$

$\vec e_y$: $m\ddot y = V_K \cdot \rho_K g - V_K \rho_{Fl} g - k_{St} \cdot v$ $\qquad v = \dot y$

Dgl.: $m\ddot y + k_{St}\,\dot y = g\, V_K (\rho_K - \rho_{Fl}) = F_0$

NR: k_{St}



$\vec F_R = -p A_K \cdot c_w \dfrac{\vec v}{|\vec v|}$

Vor: Kugelströmung $\quad 1 < Re < 1000 \quad c_w = \dfrac{24}{Re}$

$Re = \dfrac{\rho_{Fl}}{\eta} \cdot d \cdot v$

$F_R = \underbrace{\dfrac{\rho_{Fl} v^2}{2}}_{p} \cdot \dfrac{\pi}{4} d^2 \cdot \dfrac{24\eta}{\rho_{Fl} \cdot d \cdot v} = \underbrace{3\pi\eta d}_{\text{Gesetz von Stokes}} \cdot v = k_{St}\, v$

$k_{St} = 3\pi\eta d$

③ $v(t \to \infty)$ Grenzgeschwindigkeit $\lim_{t \to \infty} v(t) = v_{Gr}$ -ans.

$\rightsquigarrow \ddot{y} = 0$ in Dgl.

$$k_{St} \cdot v_{Gr} = F_0 \qquad v_{Gr} = \frac{F_0}{k_{St}} = \frac{g V_K (\rho_K - \rho_{Fe})}{3\pi \eta d}$$

NR: $V_K = \frac{\pi}{6} d^3$

$$v_{Gr} = \frac{g \cancel{\pi} d^{\cancel{3}\,2} (\rho_K - \rho_{Fe})}{18 \cancel{\pi} \eta \cancel{d}} = \frac{g d^2}{18 \eta} (\rho_K - \rho_{Fe})$$

Lsg: $m \dot{v} + k_{St} v = F_0 \qquad \dot{v} = \frac{dv}{dt} = \frac{1}{m} (F_0 - k_{St} \cdot v)$

$$\frac{1}{F_0 - k_{St} v} dv = \frac{1}{m} dt \;\Big|\int \qquad \int \frac{1}{a - bx} dx = -\frac{1}{b} \ln(a - bx)$$

$$-\frac{1}{k_{St}} \ln(F_0 - k_{St} v) = \frac{1}{m} t + C_1 \qquad \text{AB: } v(t=0) = 0$$

$$-\frac{1}{k_{St}} \ln(F_0) = C_1$$

$$-\frac{1}{k_{St}} \ln(F_0 - k_{St} v) = \frac{1}{m} t - \frac{1}{k_{St}} \ln(F_0) \to v(t)$$

$$\ln(F_0 - k_{St} v) = -\frac{k_{St}}{m} t + \ln(F_0)$$

$$F_0 - k_{St} v = e^{-\frac{k_{St}}{m} t} \cdot F_0$$

$$v(t) = \frac{-F_0 e^{-\frac{k_{St}}{m} t} + F_0}{k_{St}} = \frac{F_0}{k_{St}} \left(1 - e^{-\frac{k_{St}}{m} t}\right)$$

Grenzwert $\lim_{t \to \infty} v(t) = \frac{F_0}{k_{St}} = v_{Gr}$