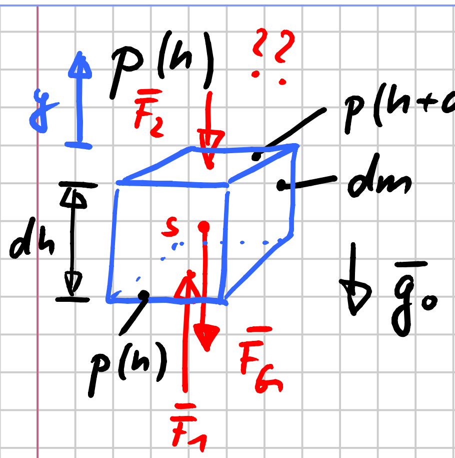
Barometrische Höhenformel

Prof. Grabow



$$p(h+dh) = p(h) + dp$$

$$\bar{F}_G = dm\,\bar{g}_0\,(-\bar{e}_y) = -\varrho_L\,dV\,g_0\,\bar{e}_y$$

$$= -\varrho_L A \cdot dh\, g_0\,\bar{e}_y$$

$$\bar{F}_1 = p(h)\cdot A\,\bar{e}_y$$

$$\bar{F}_2 = -(p(h) + dp)\,A\,\bar{e}_y$$

Kräftegleichgewicht: $\bar{F}_1 + \bar{F}_2 + \bar{F}_G = \bar{0}$

$\bar{e}_y$: $p(h)A - (p(h)+dp)A - \varrho_L A\,dh\,g_0 = 0 \quad \Big|\div A$

$$-dp = \varrho_L\,dh\,g_0$$

$$\frac{dp}{dh} = -\varrho_L\,g_0$$

ϱ_L ersetzen

$$pV = n\cdot RT = mR_sT$$

$$p\frac{m}{\varrho} = mR_sT$$

$$\varrho = \frac{p}{R_sT}$$

$$\frac{dp}{dh} = -\frac{p}{R_sT}\,g_0$$

$$\frac{1}{p}\,dp = -\frac{g_0}{R_sT}\,dh \quad \Big|\cdot\int$$

$$\int_{p(h_0)}^{p(h)} \frac{1}{p}\,dp = -\frac{g_0}{R_s}\int_{h_0}^{h} \frac{1}{T}\,dh$$

Frage: $T(h) = ?$

<u>Vor:</u> isotherme Atmosphäre

$$\underline{T(h) = T_0 = \text{const}}$$

$$\int_{p(h_0)}^{p(h)} \frac{1}{p}\,dp = -\frac{g_0}{R_sT_0}\int_{h_0}^{h} dh$$

$$\ln\left[\frac{p(h)}{p(h_0)}\right] = -\frac{g_0}{R_sT_0}(h-h_0)$$

$$p(h) = p(h_0)\,e^{-\frac{g_0}{R_sT_0}(h-h_0)}$$

const

$$\frac{R_sT_0}{g_0} =: h_s \quad \text{Skalenhöhe}$$

$$p(h) = p(h_0)\,e^{-\frac{(h-h_0)}{h_s}}$$

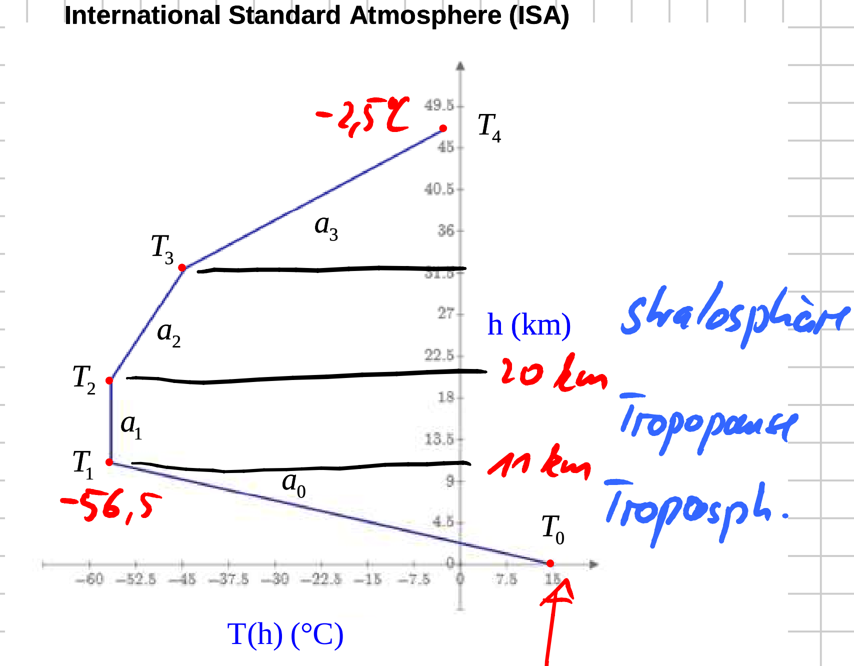
$$h_s = \frac{p_0}{\varrho_0 g_0}$$

$$\boxed{p(h) = p(h_0)\, e^{-\frac{\Delta h}{h_s}}}$$

② Lineare Temperaturänderung

$$T(h) = T_0 - a_0 (h - h_0)$$

International Standard Atmosphere (ISA)



Index	Intervall	[h] m	[T] °C	[a] K/m	[p] Pa
0	0 km – 11 km	0	15	0.0065	101325.00
1	11 km – $\overset{20}{32}$ km	11000	-56.5	0	22632.68
2	20 km – 32 km	20000	-56.5	-0.001	5475.18
3	32 km – 47 km	32000	-44.5	-0.0028	868.094
4	47 km – 52 km	47000	-2.5	0	110.92

$p(h)$: $$\int_{p(h_0)}^{p(h)} \frac{1}{p}\, dp = -\frac{M g_0}{R} \int_{h_0}^{h} \frac{1}{T(h)}\, dh$$

$$= -\frac{M g_0}{R} \int_{h_0}^{h} \frac{dh}{T_0 + a_0 h_0 - a_0 h}$$

NR: $$\int \frac{1}{b - ax}\, dx = \frac{1}{a} \ln(b - ax)$$

$$\ln\left[\frac{p(h)}{p(h_0)}\right] = \left(+\frac{M g_0}{R} \cdot \frac{1}{a_0} \ln\left(1 - \frac{a_0 \Delta h}{T_0}\right)\right)$$

$$p(h) = p(h_0)\, e^{(\,)}$$

NR: $$e^{y \ln x} = x^y$$

$$p(h) = p(h_0) \cdot \left(1 - \frac{a_0 \Delta h}{T_0}\right)^{\frac{Mg_0}{R a_0}}$$

$$\boxed{p(h) = p_n \cdot \left(1 - \frac{a_n (h - h_n)}{T_n}\right)^{\frac{Mg_0}{R a_n}}}$$