9 Modalanalyse, Eigenvektor

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Begriffsbildung: $\bar{\Psi}$ – Eigenvektor ; $\bar{\Psi} = [\Psi_1; \dots \Psi_n]^T$

$\bar{\Psi}_n$ – nte Eigenform ; $\bar{\Psi}_n = [\Psi_{1n}, \Psi_{2n}, \dots \Psi_{nn}]^T$

Bestimmung des Eigenvektors

aus Ansatz $(A - \lambda E)\bar{\Psi} \cdot e^{j\omega_0 t} = \bar{0}$

$(A - \lambda E)\bar{\Psi} = \bar{0}$ Eigenvektorgleichung

Bsp. f=2 λ_1 und λ_2 sind berechnet

① für λ_1 $(A - \lambda_1 E)\bar{\Psi}_1 = \bar{0}$ (1)

② für λ_2 $(A - \lambda_2 E)\bar{\Psi}_2 = \bar{0}$ (1)

(1) $(A - \lambda_1 E)\begin{bmatrix}\Psi_{11}\\ \Psi_{21}\end{bmatrix} = \bar{0}$

(2) $(A - \lambda_2 E)\begin{bmatrix}\Psi_{12}\\ \Psi_{22}\end{bmatrix} = \bar{0}$

(1) $\begin{bmatrix}\frac{c}{m} - \lambda_1 & -\frac{c}{2m}\\ -\frac{c}{m} & \frac{c}{m} - \lambda_1\end{bmatrix}\begin{bmatrix}\Psi_{11}\\ \Psi_{21}\end{bmatrix} = \bar{0}$ für für λ_1

$$\left.\begin{aligned}\left(\frac{c}{m} - \lambda_1\right)\Psi_{11} - \frac{c}{2m}\Psi_{21} &= 0\\ -\frac{c}{m}\Psi_{11} + \left(\frac{c}{m} - \lambda_1\right)\Psi_{21} &= 0\end{aligned}\right\} \quad \Psi_{21} = \sqrt{2}\cdot\Psi_{11}$$ linear abhängig!

z.B.: $\Psi_{11} = 1 \;\curvearrowright\; \Psi_{21} = \sqrt{2}$ $\quad \bar{\Psi}_1 = k_1\begin{bmatrix}1\\ \sqrt{2}\end{bmatrix} = k_1 \cdot \bar{\Psi}_1^N$

(2) $\begin{bmatrix}\frac{c}{m} - \lambda_2 & -\frac{c}{2m}\\ -\frac{c}{m} & \frac{c}{m} - \lambda_2\end{bmatrix}\begin{bmatrix}\Psi_{12}\\ \Psi_{22}\end{bmatrix} = \bar{0}$

$$\left.\begin{aligned}\left(\frac{c}{m} - \lambda_2\right)\Psi_{12} - \frac{c}{2m}\Psi_{22} &= 0\\ -\frac{c}{m}\Psi_{12} + \left(\frac{c}{m} - \lambda_2\right)\Psi_{22} &= 0\end{aligned}\right\} \quad \Psi_{22} = -\sqrt{2}\cdot\Psi_{12}$$ linear abhängig!

z.B. $\Psi_{12} = 1 \;\curvearrowright\; \Psi_{22} = -\sqrt{2}$ $\quad \bar{\Psi}_2 = k_2\begin{bmatrix}1\\ -\sqrt{2}\end{bmatrix} = k_2 \cdot \bar{\Psi}_2^N$

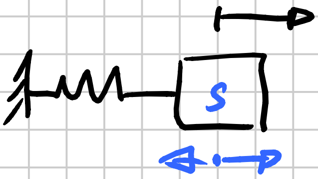
? Warum sind $\bar{\Psi}_1$ und $\bar{\Psi}_2$ beliebig normierbar ?

Ansatz: $\bar q = \bar\psi e^{j\omega_0 t} = \bar\psi_1 e^{j\omega_{01} t} + \bar\psi_2 e^{j\omega_{02} t} + \dots + \bar\psi_n e^{j\omega_{0n} t}$

physikalische Interpretation

$\bar q \mathrel{\hat=}$ Wegvektor

$\bar\psi \mathrel{\hat=}$ Wegvektoren



AB: $q(t=0) = q_0$
$\dot q(t=0) = v_0$

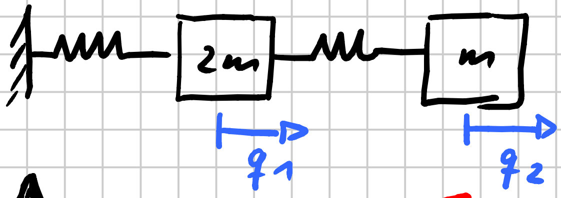
Lösung $\bar q(t)$ unter AB

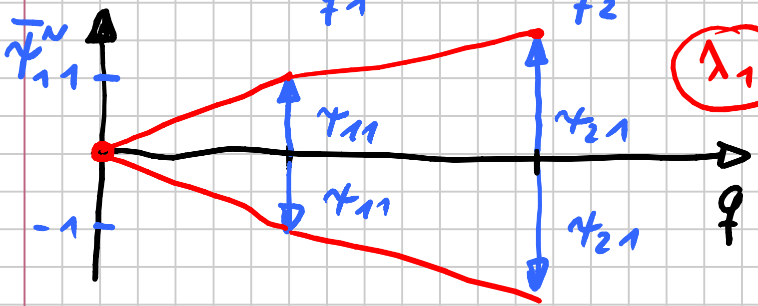
$\bar q(t) = \bar C_1 e^{j\omega_{01} t} + \dots + C_n e^{j\omega_{0n} t}$

$\bar C_1 = k_1 \bar\psi_1^N \quad ; \quad \bar C_n = k_n \bar\psi_n^N$

Grafische Darstellung der Eigenformen

Bsp. f=2



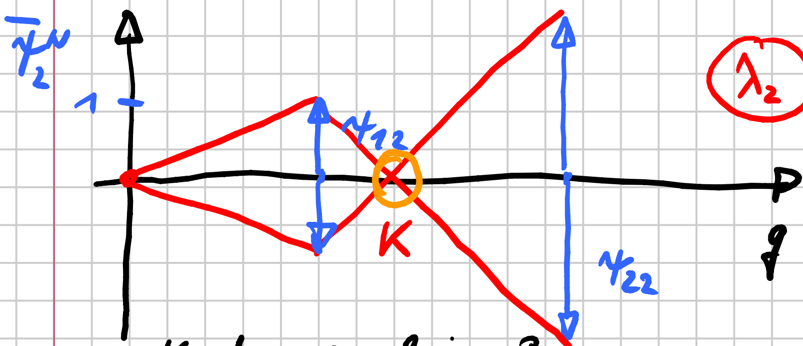


λ_1

$\psi_{11} = 1 \quad ; \quad \psi_{21} = \sqrt{2}$

$\psi_{11} = -1 \quad ; \quad \psi_{21} = -\sqrt{2}$

Knotenbild



λ_2

$\psi_{12} = 1 \quad ; \quad \psi_{22} = -\sqrt{2}$

$\psi_{12} = -1 \quad ; \quad \psi_{22} = \sqrt{2}$

Knoten K : keine Bewegung

⇝: 0 Knoten Grundwelle ⇝ $\lambda_1 \rightsquigarrow \omega_{01}$ Grundwelle

1 Knoten 1. Oberwelle ⇝ $\lambda_2 \rightsquigarrow \omega_{02}$ 1. Oberwelle

Prüfen !?

$\bar q = \sum_n \bar C_n e^{j\omega_{0n} t}$?! nicht das Knotenbild