6. Modalanalyse, Modellbildung, $f > 1$

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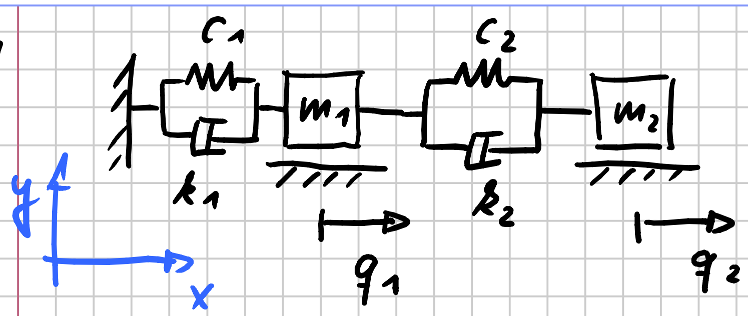
Bsp. 1



unabhängige Koordinaten

$q_1, q_2 \rightsquigarrow f = 2$

Bewegungsgleichung

q_1: $$m_1 \ddot{q}_1 = k_2(\dot{q}_2 - \dot{q}_1) - k_1 \dot{q}_1 + c_2(q_2 - q_1) - c_1 q_1 \qquad (1)$$

q_2: $$m_2 \ddot{q}_2 = -k_2(\dot{q}_2 - \dot{q}_1) - c_2(q_2 - q_1) \qquad (2)$$

(1) $$m_1 \ddot{q}_1 + (k_1 + k_2)\dot{q}_1 - k_2 \dot{q}_2 + (c_1 + c_2) q_1 - c_2 q_2 = 0$$

(2) $$m_2 \ddot{q}_2 + k_2 \dot{q}_2 - k_2 \dot{q}_1 + c_2 q_2 - c_2 q_1 = 0$$

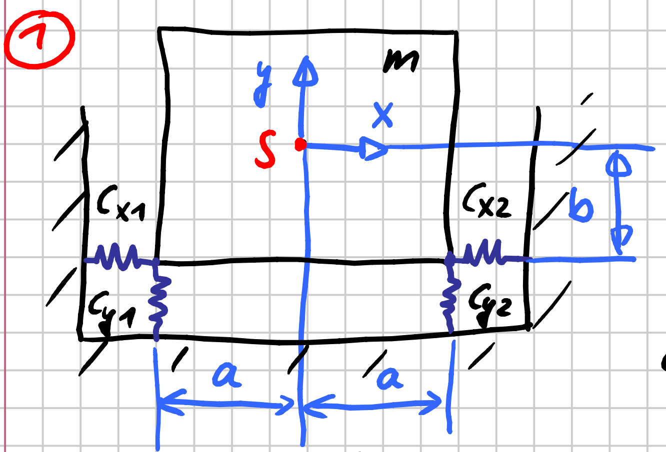
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

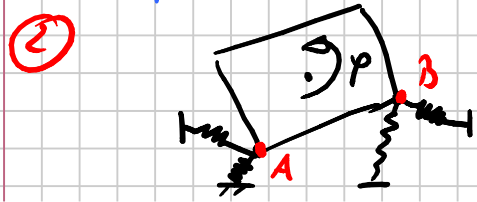
$$M \cdot \ddot{\bar{q}} + K \cdot \dot{\bar{q}} + C \cdot \bar{q} = \bar{0}$$

$$\boxed{M \ddot{\bar{q}} + K \dot{\bar{q}} + C \bar{q} = \bar{0}} \qquad f > 1$$

$$m \ddot{q} + k \dot{q} + c q = 0 \qquad f = 1$$

Bsp. 2. Blockfundament





$\bar{e}_x$: $$m\ddot{x} = -(c_{x1} + c_{x2}) x - (c_{x1} + c_{x2}) \Delta x$$

$$\Delta x = a(1 - \cos\varphi) + b \sin\varphi$$

Linearisierung $\cos\varphi \approx 1$; $\sin\varphi \approx \varphi$

$$\Delta x = b \cdot \varphi$$

$$m\ddot{x} = -(c_{x1} + c_{x2}) x - (c_{x1} + c_{x2}) b \cdot \varphi \qquad (1)$$

$\bar{e}_y$: $$m\ddot{y} = -(c_{y1} + c_{y2}) y - (c_{y2} - c_{y1}) \Delta y$$

$$\Delta y = a \sin\varphi + b(1 - \cos\varphi)$$

$$\Delta y \approx a\varphi$$

$$m\ddot{y} = -(c_{y1} + c_{y2}) - (c_{y2} - c_{y1}) a\varphi \qquad (2)$$

$\bar{e}_z$: $J_s \ddot{\varphi} = -(c_{x1}+c_{x2})\,b x - (c_{y1}-c_{y2})\,a y - (c_{x1}+c_{x2})b^2\varphi - (c_{y1}+c_{y2})a^2\varphi$ (3)

unabhängige Koordinaten:

x, y, φ $f = 3$!

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_s \end{bmatrix} \cdot \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \bar{0}$$

$$M \cdot \ddot{\bar{q}} + C \cdot \bar{q} = \bar{0}$$

$c_{11} = c_{x1} + c_{x2}$ $\quad c_{12} = 0$ $\quad c_{13} = (c_{x1}+c_{x2})b$

$c_{22} = c_{y1} + c_{y2}$ $\quad c_{21} = 0$ $\quad c_{23} = (c_{y1}-c_{y2})\cdot a = c_{32}$

$c_{33} = (c_{x1}+c_{x2})b^2 + (c_{y1}+c_{y2})a^2$ $\quad c_{31} = (c_{x1}+c_{x2})b$