4. Modalanalyse $H_R(j\Omega)$ in Residuenform

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Notiztitel 14.08.2014

$$H_R(j\Omega) = \frac{1}{m} \cdot \frac{1}{-\Omega^2 + 2D\omega_0 j\Omega + \omega_0^2}$$

N: $-\Omega^2 + 2D\omega_0 j\Omega + \omega_0^2 = f(\Omega^2)$ Substitution: $\lambda = j\Omega$, $\lambda^2 = -\Omega^2$

$$\lambda^2 + 2D\omega_0 \lambda + \omega_0^2 = f(\lambda^2) = (\lambda - \lambda_1)(\lambda - \lambda_2)$$

Nullst.: $\lambda_{1/2} = -D\omega_0 \pm \sqrt{(D\omega_0)^2 - \omega_0^2}$

$$= \underbrace{-D\omega_0}_{\delta} \pm j\underbrace{\omega_0\sqrt{1-D}}_{\omega} \qquad 0 < D < 1$$

$$\lambda_1 = -\delta + j\omega =: \lambda$$

$$\lambda_2 = -\delta - j\omega =: \lambda^*$$

Rücksubst.: $-\Omega^2 + 2D\omega_0 j\Omega + \omega_0^2 = (j\Omega - \lambda)(j\Omega - \lambda^*)$

$$H_R(j\Omega) = \frac{1}{m} \cdot \frac{1}{(j\Omega - \lambda)(j\Omega - \lambda^*)} = \frac{A}{j\Omega - \lambda} + \frac{B}{j\Omega - \lambda^*}$$

$$A = -\frac{j}{2\omega m} =: R$$

$$B = +\frac{j}{2\omega m} =: R^*$$

imaginäre Residuenform (Funktionentheorie)

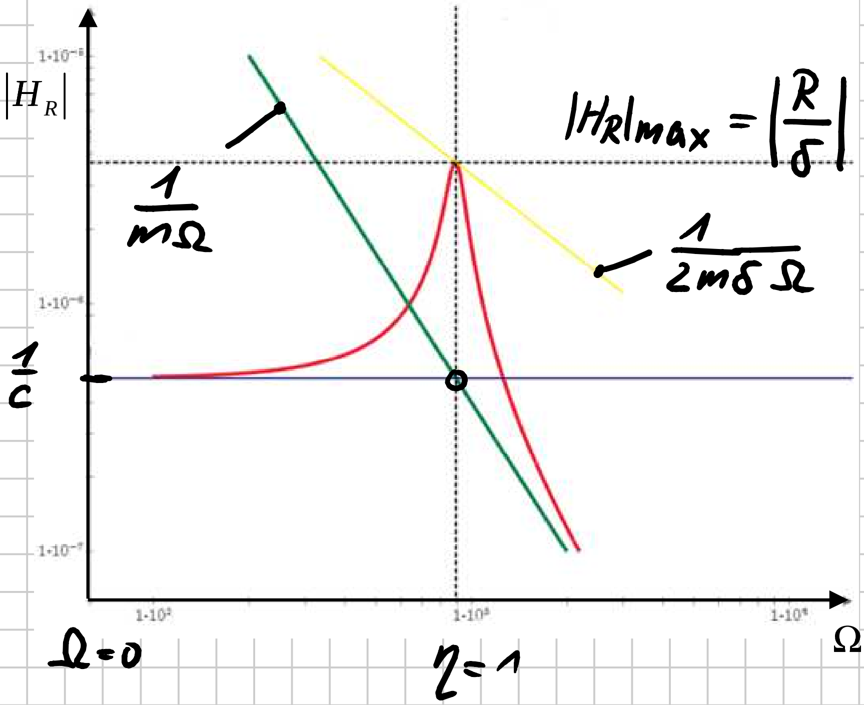
$$\boxed{H_R(j\Omega) = \frac{R}{j\Omega - \lambda} + \frac{R^*}{j\Omega - \lambda^*}}$$

reelle Residuenform (Modalanalyse)

$$R = -\frac{j}{2\omega m} \cdot \frac{j}{j} = \frac{1}{2j\omega m} \qquad R^* = \frac{j}{2\omega m} \cdot \frac{j}{j} = \frac{-1}{2j\omega m}$$

$$H_R(j\Omega) = \frac{1}{2j}\left(\frac{Rc}{j\Omega - \lambda} - \frac{Rc}{j\Omega - \lambda^*}\right); \quad Rc = \frac{1}{\omega m} \text{ (reelles Residuum)}$$

Amplitudenfrequenzgang $\left|H_R(j\Omega)\right|$



$$H_R(j\Omega) = \frac{R}{j\Omega - \lambda} + \frac{R^*}{j\Omega - \lambda^*} \qquad \lambda = -\delta + j\omega \;;\; \lambda^* = -\delta - j\omega$$

$$h_R(t) = R \cdot e^{\lambda t} + R^* e^{\lambda^* t}$$ (Sprungantwort, Übergangsfunktion $\sigma = 1$)

$$h_R(t) = R \cdot e^{-\delta t + j\omega t} + R^* e^{-\delta t - j\omega t}$$

$$= e^{-\delta t}\left[R\cos(\omega t) + jR\sin(\omega t) + R^*\cos(\omega t) - jR^*\sin(\omega t)\right]$$

$$h_R(t) = e^{-\delta t}\left[\underbrace{(R+R^*)}_{0}\cos(\omega t) + j(R-R^*)\sin(\omega t)\right]$$

NR:

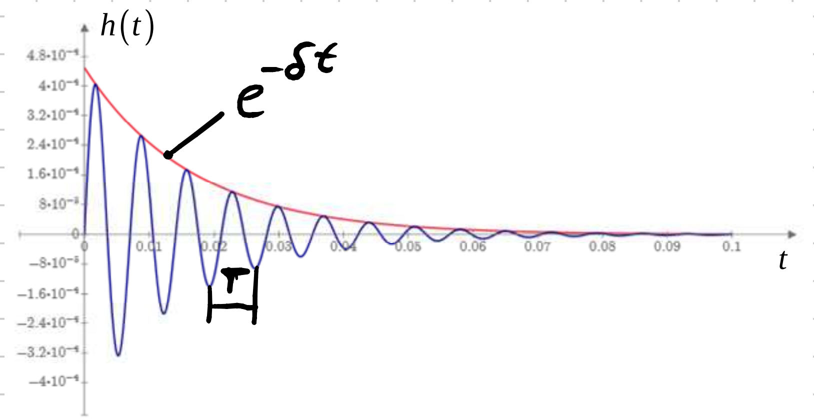
$$R + R^* = -\frac{j}{2\omega m} + \frac{j}{2\omega m} = 0$$

$$R - R^* = -\frac{j}{2\omega m} - \frac{j}{2\omega m} = -\frac{\cancel{2}j}{\cancel{2}\omega m} = -\frac{j}{\omega m}$$

$$j(R-R)^* = \frac{1}{\omega m} = 2 \cdot |R|$$

$$\boxed{h_R(t) = 2 \cdot |R| \cdot e^{-\delta t}\sin(\omega t)}$$

Sprungantwort / Übergangsfunktion



Bsp.:

System 1 $(\cdot 1000)$	System 2
$m_1 = 1\,kg$	$m_2 = 1000\,kg$
$c_1 = 900\,\frac{N}{m}$	$c_2 = 900 \cdot 10^3\,\frac{N}{m}$
$k_1 = 10\,\frac{Ns}{m}$	$k_2 = 10 \cdot 10^3\,\frac{Ns}{m}$
$\omega_{01} = \sqrt{\frac{c_1}{m_1}} = 30\,\frac{1}{s}$	$\omega_{02} = \sqrt{\frac{c_2}{m_2}} = 30\,\frac{1}{s}$
$\delta_1 = \frac{k_1}{2m_1} = 5\,\frac{1}{s}$	$\delta_2 = \frac{k_2}{2m_2} = 5 \cdot \frac{1}{s}$
$\omega_1 = \sqrt{\omega_{01}^2 - \delta_1^2} = 29.58\,\frac{1}{s}$	$\omega_2 = \sqrt{\omega_{02}^2 - \delta_2^2} = 29.58 \cdot \frac{1}{s}$

$R_1 = \frac{-j}{2\omega_1 m_1} = -j\,16.9 \cdot 10^{-3}\,\frac{m}{N \cdot s}$

$R_2 = \frac{-j}{2\omega_2 m_2} = -j\,16.9 \cdot 10^{-6}\,\frac{m}{N \cdot s}$