Modalanalyse / Übertragungsfunktion (f=1)

Prof. Grabow

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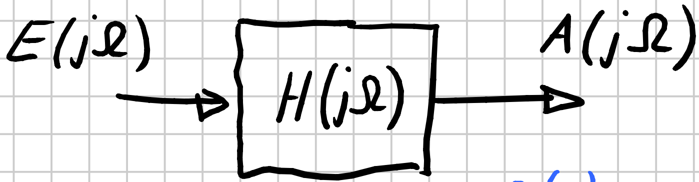
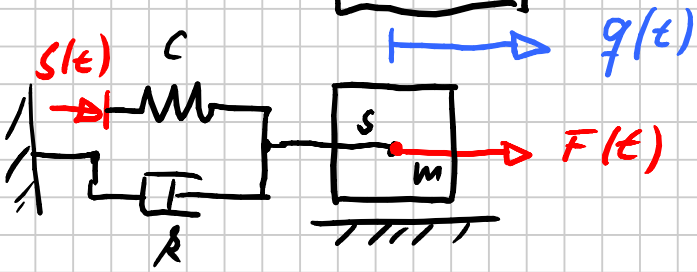
Übertragungsfunktion: $H(j\Omega)$





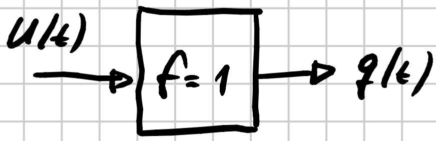
A: $q(t), \dot q(t), \ddot q(t)$

E: $F(t), s(t), \dot s(t)$

$H_1(j\Omega)$ Eingang $\hat s$; Ausgang, $q(t)$

$$q(t) = \hat q e^{j(\Omega t + \varphi)}$$

$$q(t) = \underbrace{\hat q \cos(\Omega t + \varphi)}_{Re(q(t))} + j \underbrace{\sin(\Omega t + \varphi)}_{Im(q(t))}$$



① $u(t) = \hat s \sin(\Omega t)$ indirekte Erregung (Federerregung)

Dgl: $m\ddot q + k\dot q + cq = c \cdot u(t) \quad | \div m$

$$\ddot q + 2DW_0 \dot q + \omega_0^2 q = \omega_0^2 \hat s \sin(\Omega t)$$

② $u(t) = \hat F \sin(\Omega t)$ direkte Erregung (Krafterregung)

Dgl.: $m\ddot q + k\dot q + cq = \hat F \sin(\Omega t) \quad | \div m$

$$\ddot q + 2DW_0 \dot q + \omega_0^2 q = \frac{\hat F}{m} \sin(\Omega t)$$

$$\ddot q + 2DW_0 \dot q + \omega_0^2 q = \omega_0^2 \cdot \hat s \sin(\Omega t)$$

NR: $\frac{\hat F}{m} \cdot \frac{c}{c} = \frac{c}{m} \cdot \frac{\hat F}{c} = \omega_0^2 \cdot \hat s$

Lsg.: indirekte Erregung:

Ansatz: $q_p = \hat q e^{j(\Omega t + \varphi)}$; $s(t) = \hat s \sin(\Omega t) = \hat s e^{j\Omega t}$

$$\dot q_p = j\Omega \hat q e^{j(\Omega t + \varphi)}$$

$$\ddot q_p = -\Omega^2 \hat q e^{j(\Omega t + \varphi)}$$

$$\hat q \left(-\Omega^2 + 2DW_0 j\Omega + \omega_0^2\right) e^{j\varphi} \cdot \cancel{e^{j\Omega t}} = \omega_0^2 \hat s \cancel{e^{j\Omega t}}$$

$$\frac{\hat{q}}{\hat{s}} \cdot e^{j\varphi} = \frac{\omega_0^2}{\omega_0^2 + j2D\omega_0\Omega - \Omega^2} = H_1(j\Omega)$$

$$H_1(j\Omega) = \frac{\cancel{\omega_0^2}}{\cancel{\omega_0^2}(1 + j2D\eta - \eta^2)} = \frac{1}{1-\eta^2+j2D\eta} \cdot \frac{1-\eta^2-j2D\eta}{1-\eta^2-j2D\eta}$$

$$H_1(j\Omega) = \frac{1-\eta^2-j2D\eta}{(1-\eta^2)^2+(2D\eta)^2} = \underbrace{\frac{1-\eta^2}{(1-\eta^2)^2+(2D\eta)^2}}_{Re} - j\underbrace{\frac{2D\eta}{(1-\eta^2)^2+(2D\eta)^2}}_{Im}$$

$$H_1(j\Omega) = Re\{H_1\} + Im\{H_1\}$$

$$|H_1(j\Omega)| = \sqrt{Re\{H_1\}^2 + Im\{H_1\}^2} = \frac{1}{\sqrt{(1-\eta^2)^2+(2D\eta)^2}} = V_1(D,\eta)$$

$$\sphericalangle H_1(j\Omega) = \arctan\left(-\frac{Im\{H_1\}}{Re\{H_1\}}\right) = \arctan\left(-\frac{2D\eta}{1-\eta^2}\right)$$

$|H_1(j\Omega)| = V_1(D,\eta)$ Amplitudenfrequenzgang

$\sphericalangle H_1(j\Omega) = \arctan\left(-\frac{2D\eta}{1-\eta^2}\right)$ Phasenfrequenzgang